AGATES Workshop

Tensors from the Physics Viewpoint

October 4^{th}–7^{th}, 2022

Book of Abstracts

https://agates.mimuw.edu.pl/index.php/agates/tensors-from-the-physics-viewpoint

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Talks and Abstracts

In order of appearance in the Programme of the Workshop:

On the dimension of Tensor Network Varieties

Oct 4
11:00

Alessandra Bernardi

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In this talk I will present a recent work in collaboration with C. De Lazzari and F. Gesmundo on the dimension of Tensor Network Varieties where we studied it in certain cases starting from a mathematical definition that agrees with the physical one.

Approximate Optimization with Tensor Networks

Oct 5
09:30

Bartłomiej Gardas

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Tensor networks provide a natural language to describe slightly entangled quantum computation [1]. Here, we explain how tensor based structures play an instrumental role in devising classical heuristics allowing one to calculate the low-energy spectrum of spin-glass problems (Ising models). Those physical systems of interacting spin 1/2 particles are currently being utilized to manufacture quantum annealing processors [2], i.e., specific purpose quantum computers. To raise the bar for achieving quantum supremacy with the near-term annealing technology, we have devised a deterministic quantum-inspired algorithm to efficiently sample high-quality solutions of certain spin-glass systems (including the D-Wave 2000Q annealer) that encode hard optimization problems [3]. The idea here is to employ a Projected Entangled Pair States (PEPS), to represent the Gibbs distribution of all possible configurations and efficiently contract it. With our approach, for the hardest known problems devised on the chimera graph, and for up to 2048 spins, in the order of $O(10^{10})$ high-quality solutions were found in a single run of the algorithm. This way, we were able to calculate better solutions than have ever been reported.

[1] G. Vidal, *Efficient Classical Simulation of Slightly Entangled Quantum Computations*, Phys. Rev. Lett. **91**, 147902 (2003).

[2] A. D. King, S. Suzuki, S., J. Raymond, et al. *Coherent Quantum Annealing in a Programmable* 2000 *Qubit Ising Chain*, Nat. Phys. (2022).

[3] M. M. Rams, M. Mohseni, D. Eppens, K. Jałowiecki, B. Gardas. *Approximate Optimization, Sampling and Spin-Glass Droplets Discovery with Tensor Networks*, Phys. Rev. E **104**, 025308 (2021).

Oct 5
11:00

Hyperdeterminant in Fermionic Fock Space from the e8 Lie Algebra

Frédéric Holweck

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In quantum information the problem of classification of entanglement of pure multipartite quantum states is an important question that boils down to orbit classification of tensors under different group actions. Invariant polynomials can be used for instance to separate orbits and that is how Cayley's hyperdeterminant was introduced in the QI literature twenty years ago. In this talk, I'll discuss the spinorial representation of Spin(16) from the Fermionic Fock space perspective. I'll show how the embeddings of the bosonic, qubits and fermionic Hibert spaces can be interpreted in this setting and how the natural notion of discriminant of the e8 Lie algebra projects to the hyperdeterminant for the Fermionic Fock space representation of Spin(16).

This talk is based on previous works with Peter Levay and Luke Oeding.

Oct 6
09:30

Algebraic Geometry and Many Body Entanglement

Adam Sawicki

Center for Theoretical Physics of the Polish Academy of Sciences, Al. Lotników 32/46, 02-668 Warsaw, Poland

During my talk I will review results obtained by myself and my collaborators during the last 10 years regarding the correlation properties of many particle states. In particular I will focus on algebraic geometry methods. I will also give a few open problems that might be solved during the workshop.

Tropical Linear Regression and Low-Rank Approximation — a First Step in Tropical Data Analysis

Oct 6
11:00

Yang Qi

Inria Saclay-Île-de-France Research Centre, 91120 Palaiseau Cecex and CMAP, École Polytechnique, 91128 Palaiseau Cedex, France

Tropical data arise naturally in many areas, such as control theory, phylogenetic analysis, machine learning, economics, and so on. However, many fundamental problems still deserve further investigations and more powerful tools need to be developed. In this talk, as a first step in tropical data analysis, we would like to introduce two useful models, namely tropical linear regression and tropical low-rank approximation.

More precisely, for a collection V of finitely many points, the tropical linear regression problem is to find a best tropical hyperplane approximation of V. We will establish a strong duality theorem, showing the above distance coincides with the maximal radius of a Hilbert's ball contained in the tropical polyhedron spanned by V. Algorithmically speaking, this regression problem is polynomial-time equivalent to mean payoff game. As an application, we illustrate our results by solving an inverse problem from auction theory.

Another important tool we will study is tropical low-rank approximation. We will systematically discuss the relations among different notions of rank in the tropical setting. In particular, we will reveal a close relation between tropical linear regression and best rank-2 approximation, which provides us an efficient algorithm for finding a best rank-2 matrix approximation for a given matrix.

The talk is based on a joint work with Marianne Akian, Stéphane Gaubert, and Omar Saadi.

Measures of Quantum Entanglement

Oct 6
14:00
online talk

Shmuel Friedland

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One of the most fundamental notions in quantum mechanics is the notion of quantum entanglement. It deals with d-partite system for $d \geq 2$. It was discovered by Einstein, and his collaborators, and by Schrödinger around 1935. The bipartite case $d = 2$ is well understood. Mathematically, it can be stated and treated by matrices. The case $d \geq 3$ can be stated and treated by tensors. A tensor is a multi-array with $d \geq 3$ indices, which is a vector in the tensor product of d-vector spaces. The underlying difficulty of tensors is that most of the natural notions as rank, spectral and nuclear norms, that are used as measures of quantum entanglement, are NP-hard to compute.

In this talk we will survey some known results and open problems related to quantum entanglement and their computations.

[1] M. Aliabadi, S. Friedland, *On the Complexity of Finding Tensor Ranks*, Commun. Appl. Math. Comput., **3**(2), 281–289 (2021),

[2] W. Bruzda, S. Friedland, K. Życzkowski, *Tensor Rank and Entanglement of Pure Quantum States*, arXiv:1912.06854 (2022).

[3] H. Derksen, S. Friedland, L.-H. Lim, L. Wang, *Theoretical and Computational Aspects of Entanglement*, arXiv:1705.07160 (2017).

[4] S. Friedland and L.-H. Lim, *Nuclear Norm of Higher-Order Tensors*, Mathematics of Computation, **87**, 1255–1281 (2018).

[5] S. Friedland, T. Kemp, *Most Boson Quantum States are Almost Maximally Entangled*, Proceedings of Amer. Math. Soc. **146**, 5035–5049 (2018).

[6] S. Friedland, L. Wang, *Spectral Norm of a Symmetric Tensor and its Computation*, Mathematics of Computation, **89**, 2175–2215 (2020).

Oct 7
09:30

Tensor Decompositions in Classical and Quantum Informatics

Piotr Gawron

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Data often naturally come in form of arrays indexed by multiple indices. At same time, the mathematical description of quantum information also requires naturally use of arrays with multiple indices. It is interesting how one can exploit these mathematical facts and employ the notion of tensor decomposition for both data processing and understanding multipartite quantum states. During the talk we will explain the notions of tensor ranks, and tensor decompositions and show their applications in the areas of quantum information and data compression.

And... some virtual chickens will be harmed during the talk.

Persistent Tensors: Multipartite Entanglement and Algebraic Geometry

Oct 7
11:00

Masoud Gharahi

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Two central problems in entanglement theory are the classification and interconversion of entangled states under Stochastic Local Operations and Classical Communication (SLOCC). I talk about the advantage of using tools from algebraic geometry for the problems mentioned above. I construct a lower bound of the tensor rank for a class of tensors, which I call persistent tensors. Then, I present a specific subclass of persistent tensors, of which the lower bound is tight. In addition, I show that this subclass of persistent tensors is indeed a generalization of multiqubit W states within multiqudit systems (I call them multiqudit M states) and is geometrically in the orbit closure of multiqudit GHZ states. Consequently, I show that one can obtain a multiqudit M state from a multiqudit GHZ state via asymptotic SLOCC with rate one. I also present a fine-structure entanglement classification under SLOCC by algebraic geometry during this talk. Regarding the classification, I employ specific algebraic-geometry tools that are SLOCC invariants, namely, secant varieties and multilinear ranks.
